Forces, Gravity, Motion, Energy

Forces

- **Gravity**: $\vec{F}_g = G\frac{m_1 m_2}{r^2}\hat{r}$ Affects massive objects, always attractive, mitigated by the exchange of *gravitons*.
- **Coulomb**: $\vec{F}_c = k\frac{q_1 q_2}{r^2}\hat{r}$ Affects charged objects, may be attractive or repulsive, mitigated by the exchange of *photons*.
- **Strong nuclear**: Stronger than the Coulomb force at short range. Keeps the protons in the nucleus of the atom together in spite of the Coulomb repulsion
- **Weak nuclear**: Radioactive decay.

Unification of these forces (The G.U.T.) is a prime focus of research in high-energy physics.

In astronomy we deal primarily with the gravitational force.

Newton's Laws of Motion

1. An object at rest stays at rest. An object in motion in a straight line at constant speed stays in motion in a straight line at constant speed unless acted on by some outside force.
2. $\vec{F} = m\vec{a}$
3. For every force there is an equal and opposite force, i.e., action and reaction occur as a pair.

Kinematics/Dynamics – Describing Motion

Vector: a quantity with both magnitude and direction. Vectors are of enormous importance in science. Some common vectors.

- Displacement: The distance an object moves from some reference point and its direction.
- Velocity: Distance per unit time. The magnitude of the velocity vector is known as speed.
- Acceleration: Change in velocity per unit time, i.e., speeding up, slowing down or changing direction.

Centripetal Forces

When an object moves along any curved path, according to Newton's first law, it must have some outside force acting on it. In the special case where the object follows a closed path (i.e. circular or elliptical) it is said to be under the influence of a *centripetal* (center seeking) force. One complete path is often referred to as an *orbit*. This, of course, is of special interest to us in astronomy.
According to Newton's second law the magnitude of the centripetal force on an object must equal the object's mass times it's acceleration. Thus:

$$F_c = ma = m\frac{v^2}{r}$$

Some centripetal forces of interest:

- $F_c = m\frac{v^2}{r} = T$ Tension
- $F_c = m\frac{v^2}{r} = N$ Normal Force
- $F_c = m\frac{v^2}{r} = m_s N$ Static Friction
- $F_c = m\frac{v^2}{r} = G\frac{m_1 m_2}{r^2}$ Gravity
- $F_c = m\frac{v^2}{r} = k\frac{q_1 q_2}{r^2}$ Coulomb Force

At this point it may be tempting for you to conjure up a *centrifugal force* acting outward on an object in circular motion as the reaction force for whatever is causing the object to move in an orbit. *This force does not exist!* There *is* an outward reaction force, but it acts on a different part of the system. If you twirl a rock around your head on a string the centripetal force acts on the rock and the outward force acts on your hand. Similarly, the centripetal force (gravity) acting on the earth causing it to orbit the sun is paired with an outward force acting on the sun, not the earth! *The net force on an orbiting object must be centripetal* (acting inward). If the action (centripetal force) were balanced by an equal and opposite force acting outward (centrifugal) on the same object the forces would be balanced and the object would continue to move in a straight line (in accordance with Newton's first law), not a curved path.

Newton's laws apply to *inertial* (non-accelerating) frames of reference. In order to validly apply Newton's laws one must be in an inertial frame. If two observers observe the same event, on in an inertial frame and one in a non-inertial frame they might very likely draw different conclusions about the forces involved. The observer in the inertial frame is the only one who may validly apply Newton's Laws.

Kepler's Laws

- Planets move in elliptical orbits with the Sun at one focus of the ellipse.
- The orbital speed of a planet varies so that a line joining the Sun and the planet will sweep over equal areas in equal time intervals.
- The amount of time a planet takes to orbit the Sun is related to its orbit's size. The square of the period (*P*) is equal to the cube of the semimajor axis (*a*):

$$P^2 = a^3$$

Surface Gravity

Surface gravity is the acceleration that occurs on an object located on the surface of a star, planet or any other astronomical body due to the force of gravity. It is manifested as the *weight* of the object. The mathematical expression for surface gravity is:

$$g = \frac{GM}{R^2}$$

where *G*, is the gravitational constant, *M* the mass of the planet, star, etc., and *R* the distance from the surface to the center of the planet, star, etc. On Earth these values yield:

$$g_{earth} = \frac{\left(6.7\times10^{-11} m^3 \cdot s^{-2} \cdot kg^{-1}\right)\left(6.0\times10^{24} kg\right)}{\left(6.4\times10^6 m\right)^2} = 9.8 m \cdot s^2$$

For an 80kg (176 lb) person:

$$F = ma = (80kg)\left(9.8m \cdot s^2\right) = 784 kg \cdot m \cdot s^{-2} = 784N$$

So the gravity exerts a force of 784 Newtons on an 80kg person on the surface of the earth.

Escape Velocity

Escape velocity or, more properly, escape speed is the speed that an object must attain in order to completely escape the gravity of a planet or star. For any roughly spherical object escape speed is:

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

On earth this is:

$$v_{esc} = \sqrt{\frac{2\left(6.7\times10^{-11} m^3 \cdot s^{-2} \cdot kg^{-1}\right)\left(6.0\times10^{24} kg\right)}{\left(6.4\times10^6 m\right)}} = 11200 m \cdot s \approx 25{,}000 mph$$

Energy

- Mechanical energy comes in two forms: Potential and Kinetic
- Potential – energy of position
- Kinetic – Energy of motion
- Thermal, Chemical, Nuclear, Electrical, Radiant phenomena are all sources of potential or kinetic energy

Energy may chance forms but is always *conserved*

- Chemical $\rightarrow$ Electrical
- Electrical $\rightarrow$ Thermal
- Thermal $\rightarrow$ Radiant
- Mechanical $\rightarrow$ Electrical
- Nuclear $\rightarrow$ Thermal
- Kinetic $\rightarrow$ Potential
- Potential $\rightarrow$ Kinetic